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the use of very weak reducing solution. Much time and care are given to this, and an attempt is made to keep the relative brightness of the various parts the same in kind, though not in degree, as in the unreduced negative, which is constantly used for comparison during the process."

On the Verification of the Newtonian Law.

By Ernest W. Brown, F.R.S.

In the *Monthly Notices* for March and June 1897 I gave the results of an attempt to find out the degree of accuracy with which the theoretical values of the mean motions of the lunar perigee and node could be determined with the new data then available. My computations in the lunar theory had proceeded sufficiently far to enable me to give accurately the parts of these motions inclusive of the squares of the eccentricities and inclinations; the remaining parts, due to the direct action of the Sun, were estimated from Delaunay's literal expressions. Owing to the fact that some of Delaunay's higher terms were rather doubtful, that they only included two powers of m at most, and that slow convergence rendered an estimate of the remainders very doubtful, it was expedient to assign possible errors to the results quite large in comparison with the magnitudes of the terms which were estimated. They were set down in the second paper as follows :—

Mean Motions per annum.

	For the Perigee.	For the Node.
Calculated,	+146 434″·6 ±1″·8,	−69 679″·3 ±1″·1 ;
Hansen,	+146 434″·0,	−69 676″·8;
Observed,	+146 435″·6,	−69 679″·5.

It was noted that the possible error ±1″·8 prevented any new discussion in the case of the perigee, but that the smaller possible error ±1″·1 showed that Hansen's theoretical result for the node was wrong by at least 1″·4.

A short time ago my calculations in the lunar theory reached a stage which made it possible to include in the theoretical values the terms of the fourth order with respect to the eccentricities, inclination, and ratio of the parallaxes, so far as these produced a sensible effect. Although the final verifications are not yet complete, the results have been tested sufficiently to permit of their publication with confidence in their substantial

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accuracy: as far as the solar perturbations are concerned, I do not think that they can be in error by more than two-tenths of a second. It may also be added that the *estimated* corrections, contained in the second of the papers referred to, due to the ratio of the sum of the masses of the Earth and Moon to that of the Sun, have been tested by exact calculation and found to be right.

The Annual Mean Motions now obtained are—

	For the Perigee.	For the Node.
By calculation,	$+146\ 435''\cdot5\ \pm0''\cdot2$,	$-69\ 679''\cdot6\ \pm0''\cdot2$;
By observation,	$+146\ 435''\cdot6$,	$-69\ 679''\cdot5$;

an agreement between theory and observation which was hardly to be expected. The parts due to the Sun's action are now entirely deduced from my calculations, and the parts due to planetary action and the figure of the Earth are the same as given in the first of the papers referred to above, the former being obtained by Hansen and the latter by G. W. Hill. The observed values are also the same as those given there, the values being Hansen's, confirmed by Newcomb.

The chief point of interest in connection with these results is the degree of accuracy of the Newtonian law of gravitation at the distance of the Moon. In his *Astronomical Constants* Professor Newcomb stated that the outstanding differences between theory and observation in the motions of the perihelia of the planets, especially that of *Mercury*, could be well accounted for by assuming the law of gravitation to be $r^{-2-\delta}$ instead of r^{-2} (Hall's hypothesis), where $\delta = \cdot0000001574$. This value of δ would cause a correction of $1''\cdot4$ in the case of the Moon—almost exactly the difference that Hansen had found. Newcomb adds that the evidence afforded by this agreement in the case of the Moon's perigee is rendered nugatory by the difference $-2''\cdot7$ in the case of the node; a difference which the alteration in the law will not explain.

If the new theoretical values of the motions of the Moon's perigee and node are correct, the greatest difference between theory and observation is only $0''\cdot3$, making $\delta < \cdot00000004$. Such a value for δ is quite insufficient to explain the outstanding deviation in the motion of the perihelion of *Mercury*. It appears, then, that this assumption must be abandoned for the present, or replaced by some other law of variation which will not violate the conditions existing at the distance of the Moon.

Haverford College, Pa., U.S.A.:
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